PAM3012
Digital Image Processing for Radiographers

Image Enhancement in the Frequency Domain

In this lecture

- ★Filtering in the Frequency Domain
- ★Smoothing frequency domain filters
- ★Sharpening frequency domain filters

Basics of Filtering Frequency Domain

- Frequency Domain is nothing more than the space defined by values of FT & frequency variables (u, v)
- In this lecture we put some 'meaning' to the Fourier Domain

Basics of Frequency Domain Filtering

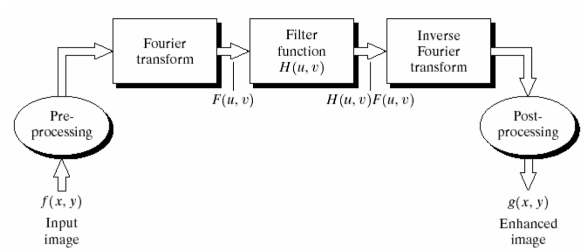
1. Compute DFT of the image, F(u, v)
2. Multiply F(u, v) by filter function, H(u, v)
3. Compute inverse DFT

Basics of Frequency Domain Filtering

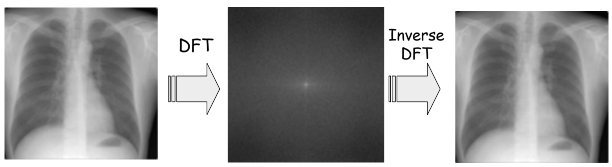
- H(u, v) is called a filter function
  - suppresses certain frequencies in transform whilst leaving others unchanged
- Analogous to coffee filter: stops larger particles whilst allowing smaller ones to pass
- Mathematically: $G(u,v) = H(u,v) \times F(u,v)$
- Filtered image obtain by taking inverse FT of G(u,v)

Basics of Frequency Domain Filtering

- Basic steps



Basics of Frequency Domain Filtering

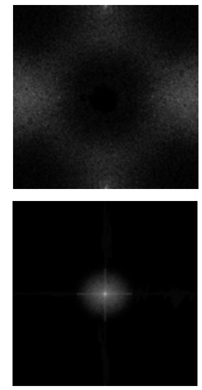


- Low frequencies: general grey-level appearance
- High frequencies: detail, edges, noise

Basics of Frequency Domain Filtering

<u>Basic Filter Functions</u>

- *Lowpass Filter:* Attenuates high frequencies 'passes' low frequencies
- *Highpass Filter:* Attenuates low frequencies 'passes' high frequencies



Basics of Frequency Domain Filtering

<u>Basic Filter Functions</u>

- *Lowpass Filter:* Attenuates high frequencies 'passes' low frequencies
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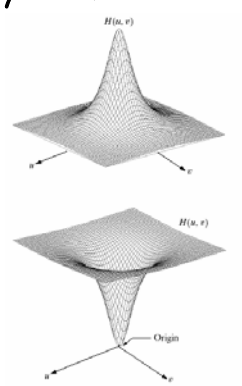
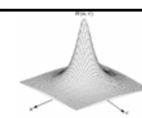
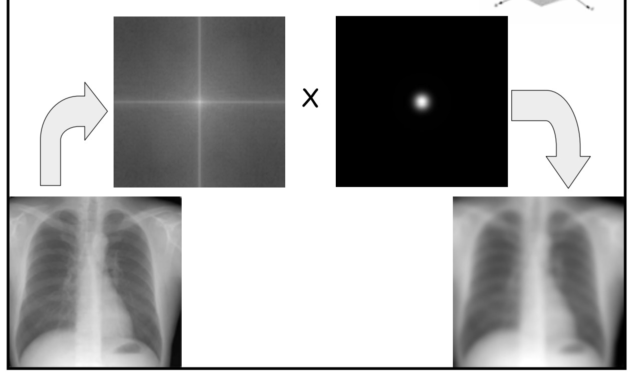
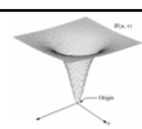


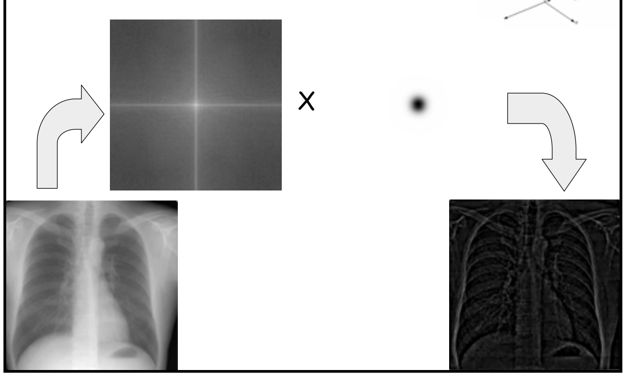
Lowpass Filter





Highpass Filter





Smoothing Frequency Domain Filters

Smoothing Frequency Domain Filters

- Edge & other sharp transitions (i.e. noise) contribute to high frequency content of Fourier Domain
- Smoothing & blurring achieved by attenuating high frequency components of Fourier Domain

$$G(u,v) = H(u,v) \times F(u,v)$$

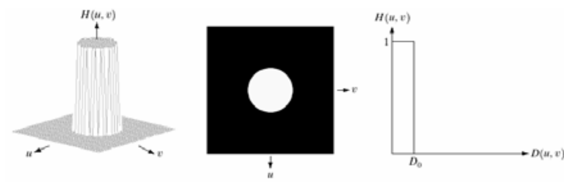
Smoothing Frequency Domain Filters

Ideal Lowpass Filter

'Cuts off' all components of FT that are greater than distance D_o from centre

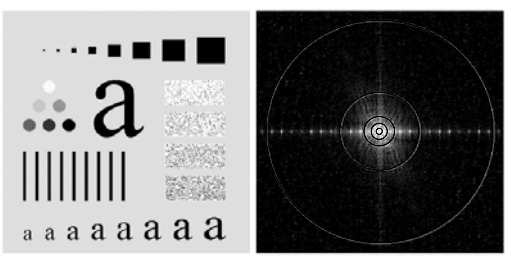
$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

Cut-off Frequency, D_0



Smoothing Frequency Domain Filters

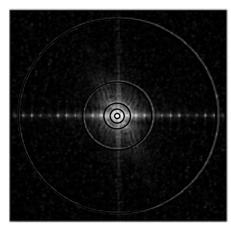
Ideal Lowpass Filter



Filter Radius

5
15
30
80
230

Ideal Lowpass Filter



Cut-off Radii

	5
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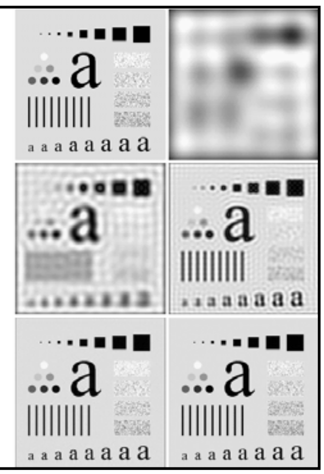


Smoothing Frequency Domain Filters

Butterworth Lowpass Filter

Transfer Function (n^{th} order)

Does not have sharp discontinuity

No clear cut-off between passed & filtered frequencies

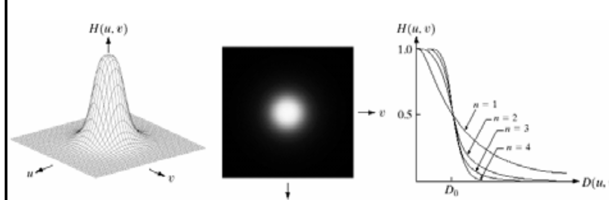
$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

Define cut-off frequency as 50% transmission

(I.e. $H(u,v) = 0.5$ when $D(u,v) = D_0$)

Smoothing Frequency Domain Filters

Butterworth Lowpass Filter



Butterworth Lowpass Filter

Order: n=2

Cut-off Radii

5

15 30

80 230

Smoothing Frequency Domain Filters

Gaussian Lowpass Filter

Does not have sharp discontinuity

No clear cut-off between passed & filtered frequencies

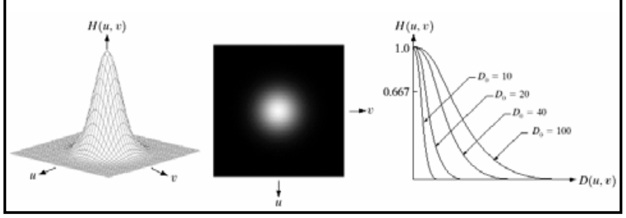
Transfer Function

$$H(u,v) = e^{-\frac{D^2(u,v)}{2D_o^2}}$$

H(u,v) = 0.607 when D(u,v) = D_0

Smoothing Frequency Domain Filters

Gaussian Lowpass Filter



Gaussian Lowpass Filter

Cut-off Radii

5

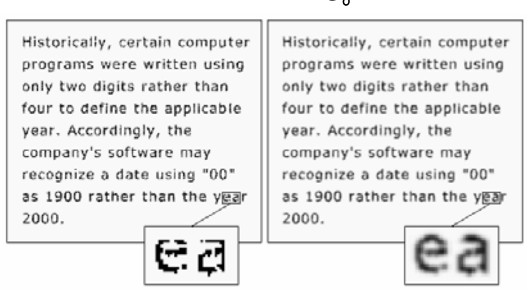
15 30

80 230

Example

Gaussian Lowpass Filter

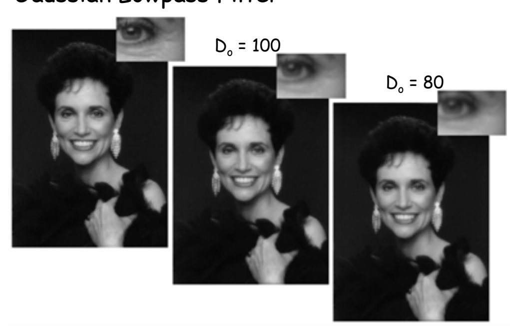
D_o = 80



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Example

Gaussian Lowpass Filter



Sharpening Frequency Domain Filters

Sharpening Frequency Domain Filters

- Smoothing & blurring achieved by attenuating high-frequency content of Fourier Transform
- Edges can be enhanced by attenuating low frequency content
- *Highpass Filtering*

Sharpening Frequency Domain Filters

- Highpass filtering is precisely reverse operation as lowpass filtering

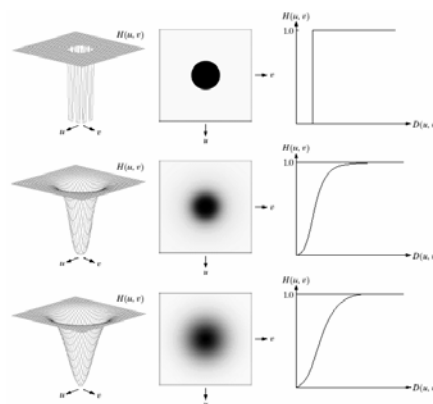
$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

Sharpening Frequency Domain Filters

Ideal Highpass

Butterworth Highpass

Gaussian Highpass



Sharpening Frequency Domain Filters

Ideal Highpass Filter

'Cuts off' all components of FT that are small than distance D_o from centre

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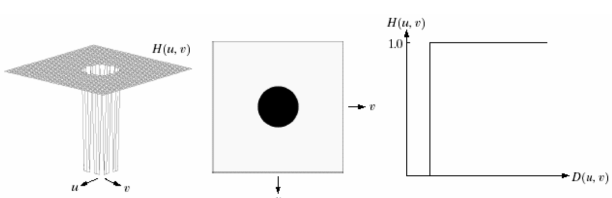
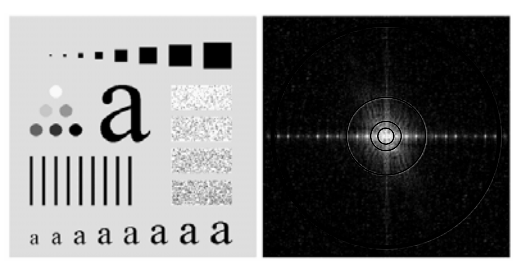
Cut-off Frequency, D_0



Sharpening Frequency Domain Filters

Ideal Highpass Filter



Filter Radius

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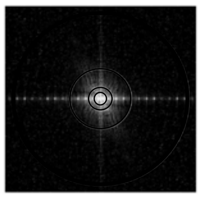
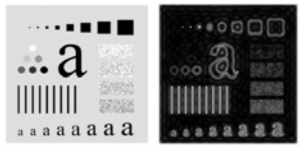
Ideal Highpass Filter





Cut-off Radii

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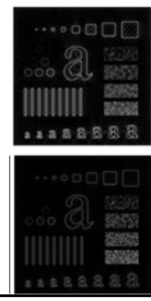

Sharpening Frequency Domain Filters

Butterworth Highpass Filter

Does not have sharp discontinuity

No clear cut-off between passed & filtered frequencies

Transfer Function (n^{th} order)

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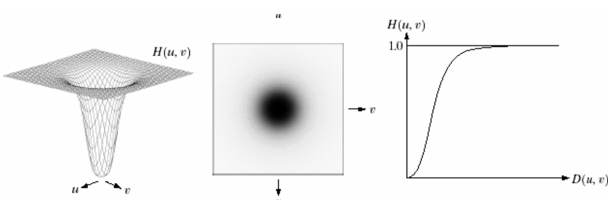
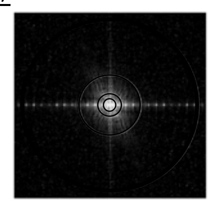
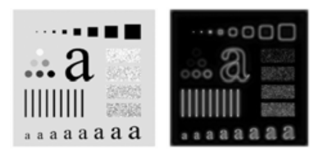
Sharpening Frequency Domain Filters

Butterworth Highpass Filter



Butterworth Highpass Filter





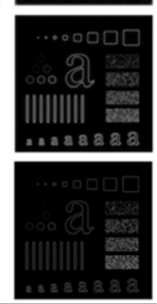
Order n=2

Cut-off Radii

15

30

80



Sharpening Frequency Domain Filters

Gaussian Highpass Filter

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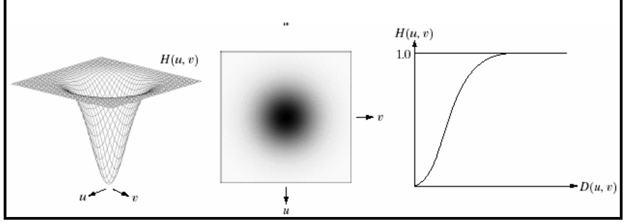
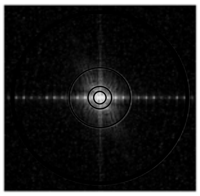
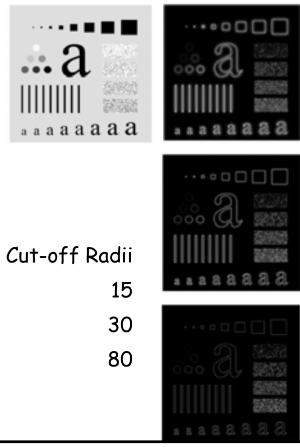
Sharpening Frequency Domain Filters

Gaussian Highpass Filter



Gaussian Highpass Filter





Cut-off Radii
15
30
80

Summary

- ★Filtering in the Frequency Domain
- ★Smoothing frequency domain filters
- ★Sharpening frequency domain filters